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ON THE MECHANISM OF NUCLEATE BOILING

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Abstract—The mechanism of nucleate boiling from a superheated surface is studied fluid-dynamically and thermodynamically.

The asymmetry of the fluid-dynamical field associated with the existence of a heating surface causes the bubbles to move away from the surface with a nearly constant speed, while the surface area of the bubbles increases linearly with respect to time. The time interval between bubble formation and departure is proportional to the third power of the radius of the bubble. The consideration of the velocity and temperature fields in the vicinity of the heating surface provides a relation between the period of the bubble cycle and the amount of superheat. The period is proportional to the third power of the radius of the bubble, when the fields interact strongly on each other, and to the second power when they do not. The radius of a bubble is inversely proportional to the amount of superheat.

NOMENCLATURE

c_p, specific heat at constant pressure;
L, latent heat of evaporation of liquid;
h, enthalpy;
H, $= L/(R_g T_e)$;
p, pressure in the fluid outside the bubble;
p_0, pressure inside the bubble;
q, heat flux through the liquid–vapour interface of the bubble;
r, radial co-ordinate;
R, distance from the spherical centre of bubble;
R_0, radius of bubble;
R_g, gas constant;
S_0, area of the liquid–vapour interface of the bubble;
t, time;
t_e, period from the departure of bubble to the new bubble formation;
t_d, period from the bubble formation to its departure;
T, temperature in the fluid outside the bubble;
T_0, temperature inside the bubble;
T_s, temperature at $z = \infty$ (saturation temperature of liquid at p_∞);
u, w, components of velocity of the fluid (Fig. 1);
V_0, volume of bubble;
z, co-ordinate perpendicular to the heating surface;
z_0, distance from the heating surface to the spherical centre of bubble;
z^*, $= 2\sigma T_s R_g/(p_\infty L)$.

Greek symbols
α, thermal diffusivity of liquid;
β, defined by equation (2.3);
γ, defined by equation (2.18);
η, ω, toroidal co-ordinates, equation (1.3);
θ, $= (T - T_s)/(T_w - T_s)$;
κ, thermal conductivity of liquid;
ϕ, velocity potential;
ν, kinematic viscosity of liquid;
ρ, density of liquid outside the bubble;
ρ_0, density of vapour inside the bubble;
σ, surface tension;
φ, angle defined in Fig. 1.

Subscripts
0, vapour inside the bubble;
d, at the departure of bubble;
e, at the generation of bubble.

INTRODUCTION

THE MECHANISM of nucleate boiling is still not well understood, despite its great technical importance and a number of experimental and theoretical works, several of which describe some features of the boiling process quite well. Observations show that the mechanism which causes the bubble to depart from the heating surface cannot be attributed only to buoyancy forces, that is, to the effect of acceleration of gravity, which has a dominant influence upon the departure process of bubbles of comparatively large radius, that is, at the lower superheat. For higher superheat, the departure process is therefore governed by a different mechanism, a fluid-dynamical and thermodynamical one, which reveals itself more evidently when the field has an asymmetrical geometry—a plane solid heating surface. To make the mechanism clear, it is necessary to analyse the process of nucleate boiling hydrodynamically and thermodynamically. Concerning nucleate boiling carried out in a superheated or subcooled liquid without any solid heating surface, a number of theoretical studies have been made recently [1–4], though the physics of practical nucleate boiling with solid heating surfaces has not yet been disclosed.

The object of the present study is to treat the process of nucleate boiling in a saturated liquid on a solid heating surface—the formation of an embryonic bubble on the heating surface, its growth and its departure from the surface—from the fluid-dynamical and thermodynamical point of view.

GROWTH AND DEPARTURE PROCESS

In the course of this process, the following assumptions are made; (1) the effects of the acceleration of. gravity, the viscosity and the compressibility of the liquid are neglected, (2) the shape of the bubble is that part of the sphere of radius R_0, with centre z_0 over the heating surface (see Fig. 1), and (3) the vapour inside the bubble is saturated corresponding the pressure inside the bubble.

First, we consider the motion of the liquid associated with the motion of the bubble—its growing in size attached to the heating surface until the departure from it. The equation of

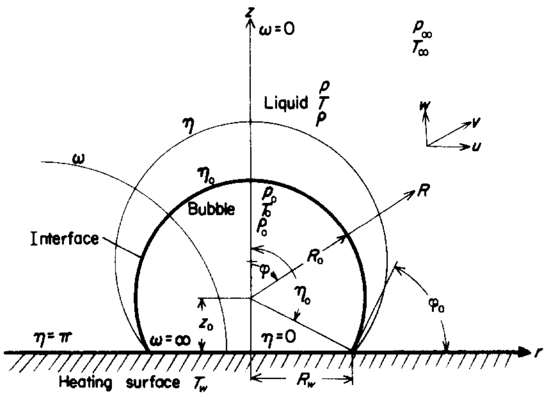

FIG. 1. Co-ordinates.

continuity concerning the liquid surrounding the bubble is

$$\frac{\partial ru}{\partial r} + \frac{\partial rw}{\partial z} = 0. \tag{1.1}$$

The boundary conditions of equation (1.1) are provided by the facts that the normal component of the velocity of the liquid vanishes on the heating surface, and the velocity of the liquid normal to the liquid–vapour interface is given as the sum of those associated with the growth of the bubble and its motion in the z-direction, namely

$$\left.\begin{aligned} w &= 0 \quad \text{at} \quad z = 0 \\ \mathbf{v}\,.\,\mathbf{R} &= \dot{R}_0 + \dot{z}_0 \cos\varphi \quad \text{at} \quad R = R_0. \end{aligned}\right\} \tag{1.2}$$

Using toroidal co-ordinates (η, ω)

$$r = R_w \frac{\sinh\omega}{\cosh\omega + \cos\eta}, \qquad z = R_w \frac{\sin\eta}{\cosh\omega + \cos\eta} \tag{1.3}$$

and the velocity potential, ϕ, defined as

$$u = \frac{\partial \phi}{\partial r}, \qquad w = \frac{\partial \phi}{\partial z} \tag{1.4}$$

we can rewrite equation (1.1) as

$$\frac{\partial}{\partial \omega}\left(\frac{\sinh \omega}{\cosh \omega + \cos \eta}\frac{\partial \phi}{\partial \omega}\right) + \frac{\partial}{\partial \eta}\left(\frac{\sinh \omega}{\cosh \omega + \cos \eta}\frac{\partial \phi}{\partial \eta}\right) = 0. \tag{1.5}$$

Putting

$$\phi = \sqrt{(\cosh \omega + \cos \eta)}\, f(\eta)\, g(\cosh \omega)$$

we obtain

$$\phi = \sqrt{(s + \cos \eta)} \int_0^\infty [A(\lambda) \sinh \lambda\eta + B(\lambda) \cosh \lambda\eta]\, P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda \tag{1.6}$$

where

$$s = \cosh \omega, \qquad P_{i\lambda - \pi/2}(s) = \frac{\sqrt{2}}{\pi} \cosh \lambda\pi \int_0^\infty \frac{\cos \lambda\zeta\, \mathrm{d}\zeta}{\sqrt{(\cosh \zeta + s)}}.$$

With the relations

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = \frac{1-s}{R_w}\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=\pi}, \qquad \left(\frac{\partial \phi}{\partial n}\right)_{R=R_0} = -\frac{1}{R_0}\frac{\sin \eta_0}{\cos \eta_0}\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=\eta_0}, \qquad \cos \varphi = \frac{1 + s \cos \eta_0}{s + \cos \eta_0}$$

the boundary conditions (1.2) are reduced to

$$\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=\pi} = 0, \qquad \left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=\eta_0} = -\frac{R_0 \cos \eta_0}{\sin \eta_0}\left[(\dot{R}_0 + \dot{z}_0 \cos \eta_0) + \frac{\dot{z}_0 \sin^2 \eta_0}{s + \cos \eta_0}\right]. \tag{1.2'}$$

If we find the solutions of equation (1.5) ϕ_1 and ϕ_2, which satisfy the following boundary conditions,

$$\left.\begin{aligned}
&\left(\frac{\partial \phi_1}{\partial \eta}\right)_{\eta=\pi} = 0, \qquad \left(\frac{\partial \phi_1}{\partial \eta}\right)_{\eta=\eta_0} = -\frac{R_0 \cos \eta_0}{\sin \eta_0}(\dot{R}_0 + \dot{z}_0 \cos \eta_0)\\
&\left(\frac{\partial \phi_2}{\partial \eta}\right)_{\eta=\pi} = 0, \qquad \left(\frac{\partial \phi_2}{\partial \eta}\right)_{\eta=\eta_0} = -\frac{R_0 \cos \eta_0}{\sin \eta_0}\frac{\dot{z}_0 \sin^2 \eta_0}{s + \cos \eta_0}
\end{aligned}\right\} \tag{1.2''}$$

the solution of equation (1.5) can be expressed as

$$\phi = \phi_1 + \phi_2.$$

With equations (1.2″) and (1.6) and the equality

$$\frac{1}{\sqrt{(s + \cos \eta)}} = \sqrt{2} \int_0^\infty \cosh \lambda\eta \operatorname{sech} \lambda\pi\, P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda,$$

ϕ_1 and ϕ_2 are given as follows:

$$\phi_1 = \sqrt{(s + \cos\eta)} \int_0^\infty \frac{R_0 \cot\eta_0 \,.\, (\dot{R}_0 + \dot{z}_0 \cos\eta_0)(\sqrt{2}) \cosh\lambda\eta_0 \operatorname{sech}\lambda\pi}{\frac{1}{2}[\sin\eta_0/(1 + \cos\eta_0)] \cosh\lambda(\eta_0 - \pi) - \lambda \sinh\lambda(\eta_0 - \pi)}$$

$$\times \cosh\lambda(\eta - \pi)\, P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda$$

$$\approx 2(\sqrt{2})\, R_0 \cot\eta_0 \,.\, (\dot{R}_0 + \dot{z}_0 \cos\eta_0) \frac{1 + \cos\eta_0}{\sin\eta_0} \sqrt{(s + \cos\eta)} \int_0^\infty \cosh\lambda\eta_0$$

$$\times \operatorname{sech}\lambda\pi \frac{\cosh\lambda(\eta - \pi)}{\cosh\lambda(\eta_0 - \pi)} P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda$$

$$\phi_2 = -\sqrt{(s + \cos\eta)} \int_0^\infty \frac{R_0 \cot\eta_0 \,.\, \dot{z}_0 \sin^2\eta_0 \,[2(\sqrt{2})/\sin\eta_0]\, \lambda \sinh\lambda\eta_0 \operatorname{sech}\lambda\pi}{\frac{1}{2}[\sin\eta_0/(1 + \cos\eta_0)] \cosh\lambda(\eta_0 - \pi) - \lambda \sinh\lambda(\eta_0 - \pi)}$$

$$\times \cosh\lambda(\eta - \pi)\, P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda$$

$$\approx -4(\sqrt{2})\, R_0 \dot{z}_0 \cot\eta_0 \,.\, (1 + \cos\eta_0) \sqrt{(s + \cos\eta)} \int_0^\infty \lambda \sinh\lambda\eta_0$$

$$\times \operatorname{sech}\lambda\pi \frac{\cosh\lambda(\eta - \pi)}{\cosh\lambda(\eta_0 - \pi)} P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda$$

Accordingly,

$$\phi(s, \eta) = 2(\sqrt{2})\, R_0 \cot\eta_0 \frac{1 + \cos\eta_0}{\sin\eta_0} \sqrt{\left(s + \cos\eta\right)}$$

$$\times \int_0^\infty [(\dot{R}_0 + \dot{z}_0 \cos\eta_0) \cosh\lambda\eta_0 - 2\dot{z}_0 \sin\eta_0 \lambda \sinh\lambda\eta_0] \operatorname{sech}\lambda\pi \frac{\cosh\lambda(\eta - \pi)}{\cosh\lambda(\eta_0 - \pi)} P_{i\lambda - \pi/2}(s)\, \mathrm{d}\lambda. \quad (1.7)$$

For $\eta \approx \eta_0$

$$\phi(s, \eta) = 2R_0 \frac{\cos\eta_0}{1 + \cos\eta_0} \left[(\dot{R}_0 + \dot{z}_0 \cos\eta_0) + \frac{\dot{z}_0 \sin^2\eta_0}{s + \cos\eta_0} \right] \bigg/ \sqrt{\left[\frac{s + \cos\eta}{s + \cos\eta_0} \right]}. \quad (1.8)$$

Next, consider the equation of motion of the liquid. Integrating the equation over the whole domain of the liquid, Σ (whose boundary is denoted as S), with the assumption that, at infinity, the velocity vanishes and the pressure approaches p_∞, yields

$$-\int_\Sigma \frac{\partial}{\partial t} \nabla\phi \, \mathrm{d}\tau + \tfrac{1}{2} \int_S \mathbf{v} \,.\, \mathbf{n}\, \mathrm{d}S + \int_S (p - p_\infty)\, \mathrm{d}S = 0$$

where $\mathbf{n}$ is a unit vector normal to the boundary. The first term in the left-hand side of the above equation can be expressed in the form of a surface integral, to which the surface of the heating wall has no contribution. Therefore, the above equation becomes

$$\frac{\partial}{\partial t} \int_{S_0} \phi_{S_0}\, \mathrm{d}S_0 + \int_{S_0} \frac{\partial \Sigma}{\partial t} \mathbf{n} \,.\, \nabla\phi \, \mathrm{d}S_0 + \tfrac{1}{2} \int_{S_0} \mathbf{v} \,.\, \mathbf{n}\, \mathrm{d}S_0 + \int_{S_0} (p - p_\infty)_{S_0}\, \mathrm{d}S_0 = 0 \quad (1.9)$$

where S_0 is the area of the liquid–vapour interface of the bubble. Equation (1.9) is one of the equations of motion of the bubble. Since the boundary condition for the equation of motion of liquid is given by

$$p_0 = \frac{1}{S_0}\int_{S_0} p_{S_0}\,\mathrm{d}S_0 + \frac{2\sigma}{R_0} \tag{1.10}$$

the fourth term in the left-hand side of equation (1.9) is

$$\left(p_0 - p_\infty - \frac{2\sigma}{R_0}\right)S_0.$$

The consideration of the motion of the bubble in the z-direction yields the other equation of motion of the bubble as

$$\tfrac{1}{2}M\frac{\mathrm{d}\dot{z}_0^2}{\mathrm{d}t} = -\int_{S_0} p\,\mathbf{v}\,.\,\mathbf{n}\,\mathrm{d}S_0 + \dot{z}_0\left[(p_0 - p_\infty)\pi R_w^2 - \sigma\sin\varphi_0\,2\pi R_w\right] \tag{1.11}$$

where M is the mass of the bubble, p_0 the pressure inside the bubble, σ the surface tension of liquid. With the relations

$$\int_{S_0} p\,\mathbf{v}\,.\,\mathbf{n}\,\mathrm{d}S_0 = \int_{\Sigma}\frac{\partial}{\partial t}\left(\frac{\rho\mathbf{v}^2}{2}\right)\mathrm{d}\tau, \qquad \int_{\Sigma}\frac{\rho\mathbf{v}^2}{2}\,\mathrm{d}\tau = -\frac{\rho}{2}\int_{S_0}\left(\phi\frac{\partial\phi}{\partial n}\right)_{S_0}\mathrm{d}S_0$$

and the assumption $\rho_0 \ll \rho$, equation (1.11) becomes

$$\frac{\partial}{\partial t}\int_{S_0}\left(\phi\frac{\partial\phi}{\partial n}\right)_{S_0}\mathrm{d}S_0 + \int_{S_0}\mathbf{v}^2\frac{\partial\sum}{\partial t}\,\mathrm{d}S_0 = \frac{2}{\rho}\dot{z}_0\left(p_\infty + \frac{2\sigma}{R_0} - p_0\right)\pi R_w^2. \tag{1.12}$$

From equation (1.7)

$$\left.\begin{aligned} \phi_{R=R_0} &= -\frac{2R_0z_0}{R_0+z_0}\left(\dot{R}_0 + \dot{z}_0\frac{z-z_0}{R_0}\right) \\ \left[\left(\frac{\partial\phi}{\partial t}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right]_{R=R_0} &= \dot{R}_0^2 + \dot{z}_0^2 + 2\frac{\dot{R}_0\dot{z}_0}{R_0}(z-z_0) \\ \left(\frac{\partial\phi}{\partial n}\right)_{R=R_0} &= \dot{R}_0 + \dot{z}_0\frac{z-z_0}{R_0} \end{aligned}\right\} \tag{1.13}$$

Substituting equation (1.13) and the relation

$$\frac{\partial\sum}{\partial t}\,\mathrm{d}S_0 = \frac{\partial\pi[R_0^2 - (z-z_0)^2]}{\partial t}\,\mathrm{d}z$$

into equations (1.9) and (1.12), we obtain

$$\begin{aligned} -4\frac{\mathrm{d}}{\mathrm{d}t}\left[R_0z_0\left(R_0\dot{R}_0 + \frac{R_0 - z_0}{2}\dot{z}_0\right)\right] + 2\left[\dot{R}_0^2 - \frac{\dot{z}_0^2}{3}\frac{R_0^2 - R_0z_0 + z_0^2}{R_0^2}\right]R_0(R_0+z_0) \\ + \left[\dot{R}_0^2 + \dot{z}_0^2 + \dot{R}_0\dot{z}_0\frac{R_0 - z_0}{R_0}\right]R_0(R_0+z_0) = \frac{2}{\rho}\left[p_\infty + \frac{2\sigma}{R_0} - p_0\right]R_0(R_0+z_0) \end{aligned} \tag{1.14}$$

$$-4\frac{d}{dt}[z_0\{R_0^2\dot{R}_0^2 + R_0\dot{R}_0\dot{z}_0(R_0 - z_0) + \tfrac{1}{3}\dot{z}_0^2(R_0^2 - R_0z_0 + z_0^2)\}]$$

$$+2R_0\dot{R}_0(\dot{R}_0^2 + \dot{z}_0^2)(R_0 + z_0) + (3\dot{R}_0^2\dot{z}_0 + \dot{z}_0^3)(R_0^2 - z_0^2) + \tfrac{4}{3}\frac{\dot{R}_0\dot{z}_0^2}{R_0}(R_0^3 + z_0^3)$$

$$= \frac{2}{\rho}\left[p_\infty + \frac{2\sigma}{R_0} - p_0\right]\dot{z}_0(R_0^2 - z_0^2). \tag{1.15}$$

The pressure inside the bubble, p_0, which is involved in equations (1.14) and (1.15) can be related to the temperature inside the bubble, T_0, by the assumption (3) which implies the condition of saturation of the vapour inside the bubble, that is, the Clausius–Clapeyron relation

$$\ln\frac{p_0}{p_e} = \frac{L}{R_g}\left(\frac{1}{T_e} - \frac{1}{T_0}\right)$$

where p_e and T_e are the pressure and the temperature of the vapour inside an embryonic bubble, respectively, at the time of the bubble initiation ($t = 0$), L latent heat of vaporization, and R_g gas constant of the vapour. Since $|T_e - T_0| \ll T_0$, the above equation is rewritten as

$$\frac{p_0 - p_e}{p_e} = \frac{L}{R_gT_e}\frac{T_0 - T_e}{T_e}. \tag{1.16}$$

The temperature of the vapour inside the bubble, T_0, is given by the relation of the energy transfer through the interface of the bubble. If the growth rate of the bubble is large compared with the rate of heat conduction in the vicinity of the interface, the variation in the temperature field of the liquid associated with the growth of the bubble should be confined within a narrow layer near the interface. Isshiki [5] observed these features of the temperature field by an optical method, so that the treatment similar to that carried out by Plesset–Zwick [6, 7] for the case of the growth of a bubble without any solid heating surface can be used to obtain the temperature field of the liquid surrounding the bubble.

Since we can usually assume that the thickness of the temperature boundary layer on the liquid–vapour interface of the bubble is so thin compared with the radius of the bubble that the curvature of the liquid–vapour interface should hardly affect the temperature field, the equation of energy with the co-ordinate, χ, whose origin is on the liquid–vapour interface, becomes

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T}{\partial t} = 0 \tag{1.17}$$

whose boundary conditions are

$$\frac{\partial T}{\partial x} = f(t) \quad \text{at} \quad x = 0,$$

$$T = T_\infty(z) \quad \text{at} \quad x = \infty \tag{1.18}$$

where $f(t)$ denotes a quantity proportional to the amount of heat flow through the liquid–vapour interface into the bubble, defined as

$$q = \int_0^{\varphi_0} \kappa f(t)\, 2\pi R_0^2 \sin\varphi \, d\varphi = \kappa f(t)\, S_0 \tag{1.19}$$

where κ is the thermal conductivity of the liquid. $T_\infty(z)$ is the temperature at the point far from the liquid–vapour interface and, with the assumption of a thin boundary layer of temperature, can be replaced by the temperature at the point on the plane parallel to the heating surface and far from the liquid–vapour interface, that is, from equation (2.14),

$$T_\infty(z) = T_w - (T_w - T_s)(2/\sqrt{\pi}) \int_0^{z/\{2\sqrt{[\alpha(t_e+t)]}\}} \times \exp[-\xi^2]\, d\xi \tag{1.20}$$

where T_w is the temperature of the heating surface, T_s that of liquid at $z = \infty$, that is, the saturation temperature corresponding to the pressure p_∞, and t_e the time interval from the departure of the bubble to the generation of the next embryonic bubble.

Taking the Laplace transform of $T, f(t)$ and T_∞, denoted as Θ, $F(s)$ and Θ_∞, respectively, we obtain from equation (1.17)

$$\Theta(x,s = -\sqrt{(\alpha/s)}\,F(s)\exp\left[-\sqrt{(s/\alpha)}x\right] + \Theta_\infty(s).$$

Accordingly, the temperature of the liquid in the vicinity of the bubble is

$$T(x,t) = -\sqrt{(\alpha/\pi)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{1}{2}}} \times \exp\left[-x^2/4\alpha(t-\tau)\right]\mathrm{d}\tau + T_\infty(z) \qquad (1.21)$$

and the temperature of the liquid at the liquid–vapour interface is

$$T(0,t) = T_\infty(z) - \sqrt{(\alpha/\pi)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{1}{2}}}\mathrm{d}\tau. \qquad (1.22)$$

If a mean value of the above obtained temperature, an area-mean for simplicity, is used for the temperature of the vapour inside the bubble, the latter becomes

$$T_0 = \frac{1}{R_0+z_0}\int_0^{R_0+z_0} T_\infty(z)\,\mathrm{d}z - \sqrt{(\alpha/\pi)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{1}{2}}}\mathrm{d}\tau. \qquad (1.23)$$

Since $T_\infty(z)$ given by equation (1.20) is very nearly equal to T_s for $z \geqq 4\sqrt{[\alpha(t_e+t)]}$, the first term in the right-hand side of equation (1.23) can be reduced to

$$\int_0^{R_0+z_0} T_\infty(z)\,\mathrm{d}z = T_s(R_0+z_0-z_s) + \int_0^{z_s} T_\infty(z)\,\mathrm{d}z$$

where $z_s = 4\sqrt{[\alpha(t_e+t)]}$. Furthermore, since the second term in the right-hand side of the above equation is nearly equal to $(T_w + T_s)z_s/2$, we obtain

$$\int_0^{R_0+z_0} T\ (z)\,\mathrm{d}z \approx T_s(R_0+z_0) + (T_w - T_s)z_s/2.$$

Considering this relation and $f(t) = 0$ for $z > z_s$ yields

$$T_0 = T_s + \frac{z_s}{R_0+z_0}\left[\tfrac{1}{2}(T_w - T_s) - \sqrt{(\alpha/\pi)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\frac{1}{2}}}\mathrm{d}\tau\right] \qquad (1.24)$$

for $R_0 + z_0 > z_s$.

The rate of heat flow per unit time through the liquid–vapour interface into the bubble, q, is given by

$$q = \frac{\mathrm{d}}{\mathrm{d}t}(Mh_0) - \frac{\mathrm{d}M}{\mathrm{d}t}h - \frac{\mathrm{d}}{\mathrm{d}t}\left(V_0\frac{2\sigma}{R_0}\right) + \frac{\mathrm{d}E_f}{\mathrm{d}t} \qquad (1.25)$$

in which the contribution of the direct heat flow through the solid–vapour interface into the bubble is neglected. h_0 and h are enthalpies per unit mass of the vapour inside the bubble and of the liquid outside, respectively, and V_0 is the volume of bubble. E_f denotes the surface energy of the liquid–vapour interface of the bubble and is given by the following relation

$$\frac{\mathrm{d}E_f}{\mathrm{d}t} = \left(\sigma - T\frac{\mathrm{d}\sigma}{\mathrm{d}T}\right)\frac{\mathrm{d}S_0}{\mathrm{d}t}.$$

V_0 and S_0 are obtained from Fig. 1 as

$$V_0 = \frac{\pi}{3}(R_0+z_0)^2(2R_0 - z_0),$$

$$S_0 = 2\pi R_0(R_0+z_0). \qquad (1.26)$$

Neglecting the effect of temperature upon the surface tension, we obtain from equations (1.19) and (1.25)

$$f(t) = \frac{1}{\kappa S_0}\left\{\rho_0 V_0\frac{\mathrm{d}h_0}{\mathrm{d}t} + L\frac{\mathrm{d}(\rho_0 V_0)}{\mathrm{d}t} - 2\sigma\left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{V_0}{R_0}\right) - \frac{1}{2}\frac{\mathrm{d}S_0}{\mathrm{d}t}\right]\right\}. \qquad (1.27)$$

Usually, the first and third terms in the right-hand side of the above equation are small enough to be neglected compared with the second term, where

$$f(t) = \frac{L}{\kappa S_0}\frac{\mathrm{d}(\rho_0 V_0)}{\mathrm{d}t}. \qquad (1.27')$$

Now, all quantities in the process of growth and departure are predicted; the temperature inside the bubble, T_0, is obtained from the equation of energy of the bubble, (1.23) or (1.24), the pressure inside the bubble, p_0, from the condition of the saturation of vapour (1.16) and R_0 and z_0 from the equations of motion in the r- and z-directions, (1.14) and (1.15).

Let us consider the process shortly after the generation of an embryonic bubble. Expanding R_0 and z_0 with respect to time, we obtain

$$\left.\begin{aligned} R_0 &= R_e + R_1 t + R_2 t^2 + \ldots. \\ z_0 &= z_e + z_1 t + z_2 t^2 + \ldots. \end{aligned}\right\} \quad (1.28)$$

R_e and z_e, and R_1 and z_1 have the following relations from equations (2.39) and (2.40) which are to be mentioned later.

$$z_e = R_e \cos\varphi_e, \qquad z_1 = R_1 \cos\varphi_e.$$

The same procedure of expansion of T_0 and p_0 gives

$$T_0 = T_e(1 + c_1 t + c_2 t^2 + \ldots) \qquad (1.29)$$

$$p_0 = p_e(1 + c_1 H t + c_2 H t^2 + \ldots) \qquad (1.30)$$

where $H = L/(R_g/T_e)$. Assuming that the vapour inside the bubble should obey the law of state of an ideal gas, we obtain from equations (1.29) and (1.30)

$$\rho_0 = \rho_e[1 + c_1 H t + \{c_2(H-1) - c_1^2 H\} t^2 + \ldots]. \quad (1.31)$$

Substituting equations (1.28) to (1.31) into equations (1.23) and (1.27) yields c_1, c_2, etc., as functions of R_e, R_1, R_2, z_2, etc.; for example,

$$c_1 = -\frac{2\sigma}{R_e p_e}\frac{1}{H}\frac{R_1}{R_e}. \qquad (1.32)$$

Substituting equation (1.28) into equations (1.14) and (1.15) gives R_2, z_2, R_3, z_3, etc., as

$$\left.\begin{aligned} R_2 &= a_2\frac{R_1^2}{R_e}, & R_3 &= a_3\frac{R_1^3}{R_e^2}\ldots \\ z_2 &= b_2\frac{R_1^2}{R_e}, & z_3 &= b_3\frac{R_1^3}{R_e^2},\ldots \end{aligned}\right\} \quad (1.33)$$

where a_2 and b_2 are the roots of

$$A_r a_2 + B_r b_2 = C_r$$

$$A_z a_2 + B_z b_2 = C_z$$

in which

$$A_r = 2$$

$$B_r = 1 - \cos\varphi_e$$

$$C_r = \frac{1}{2\cos\varphi_e}[\tfrac{3}{2} - 4\cos\varphi_e - \tfrac{17}{6}\cos^2\varphi_e - 3\cos^3\varphi_e - m_e(1+\cos\varphi_e)]$$

$$A_z = 2 + \cos\varphi_e - \cos^2\varphi_e$$

$$B_z = 1 - \tfrac{1}{3}\cos\varphi_e - \tfrac{2}{3}\cos^2\varphi_e + \tfrac{2}{3}\cos^3\varphi_e$$

$$C_z = \frac{1}{8\cos\varphi_e}[2 - 7\cos\varphi_e - \tfrac{26}{3}\cos^2\varphi_e + 8\cos^3\varphi_e + 4\cos^4\varphi_e - \tfrac{11}{3}\cos^5\varphi_e - m_e\cos\varphi_e(1-\cos^2\varphi_e)]$$

and a_3 and b_3 are the roots of

$$A_r' a_3 + B_r' b_3 = C_r'$$

$$A_z' a_3 + B_z' b_3 = C_z'$$

in which A_r', A_z', B_r' and B_z' are functions of $\cos\varphi_e$, and C_r' and C_z' are functions of a_2, b_2, m_e, m_1 and $\cos\varphi_e$. m_e and m_1 are the coefficients of expansion

$$\frac{2}{\rho}\left(p_\infty + \frac{2\sigma}{R_0} - p_0\right) = m_e R_1^2 + m_1\frac{R_1^3}{R_e}t + \ldots, \qquad (1.34)$$

that is,

$$m_e = \frac{2}{\rho}\frac{1}{R_1^2}\left(p_\infty + \frac{2\sigma}{R_e} - p_e\right)$$

$$m_1 = -\frac{2}{\rho}\frac{1}{R_1^2}\frac{R_e}{R_1}\left(c_1 H p_e + \frac{2\sigma}{R_e}\frac{R_1}{R_e}\right).$$

For the case of nucleate boiling of saturated pure water at atmospheric pressure with $\cos \varphi_e = 0.5$, since $m_e \ll 1$ and $m_1 \approx 0$,

$$\left.\begin{aligned} R_0 &= R_e + R_1 t - 0.834 \frac{R_1^2}{R_e} t^2 + 0.232 \frac{R_1^3}{R_e^2} t^3 \ldots \\ z_0 &= 0.5\, R_e + 0.5\, R_1 t + 1.668 \frac{R_1^2}{R_e} t^2 - 0.250 \frac{R_1^3}{R_e^2} t^3 + \ldots \end{aligned}\right\} \quad (1.35)$$

these are illustrated in Fig. 2. It is shown from the figure that the asymmetry of the fluid-dynamical field attributed to the existence of a solid heating surface, which is manifested in equations (1.14) and (1.15), should cause the bubble to move away from the heating surface immediately after its embryonic generation. Figure 3 shows the temperature and pressure inside the bubble shortly after formation.

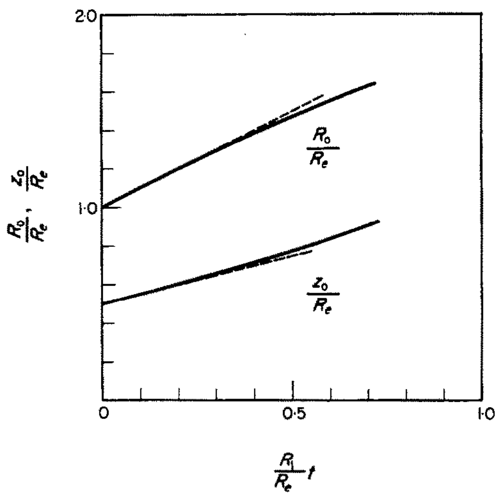

FIG. 2. Growth and departure motion shortly after bubble generation.

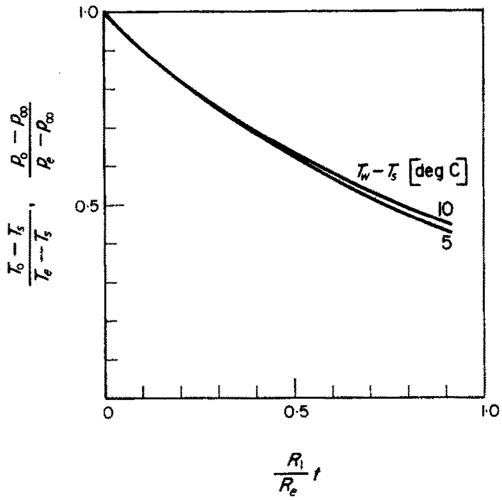

FIG. 3. Temperature and pressure inside a bubble shortly after generation.

Next, let us consider the process immediately before the departure, $t \approx t_d$. At the time near to that of the departure, the temperature inside the bubble, T_0, becomes nearly equal to the saturation temperature, T_s, so that we obtain from equation (1.25)

$$q \approx L\rho_0 \dot{V}_0$$

and from equation (1.27)

$$f(t) \approx \frac{L\rho_0}{\kappa} \dot{R}_0.$$

Substituting the above equation into equation (1.24) and putting $T_0 = T_s$ yield

$$\tfrac{1}{2}(T_w - T_s) = \sqrt{\left(\frac{\alpha}{\pi}\right)} \frac{L\rho_0}{\kappa} \int_0^t \frac{\dot{R}_0(\tau)}{(t - \tau)^{\frac{1}{2}}} \, d\tau. \quad (1.36)$$

In order that the right-hand side of equation (1.36) should be independent of time, it is required that

$$R_0 = R_d \sqrt{(t/t_d)} \quad (1.37)$$

where

$$R_d = \frac{T_w - T_s}{\sqrt{(\pi\alpha)}} \frac{\kappa R_g T_s}{L p_\infty} \sqrt{(t_d)}. \quad (1.38)$$

On the other hand, since the right-hand sides of equations (1.14) and (1.15) vanish at $t \approx t_d$, putting

$$R_0 = R_d\left(\frac{t}{t_d}\right)^m, \quad z_0 = R_d\left(\frac{t}{t_d}\right)^n \tag{1.39}$$

and neglecting the right-hand sides of those equations yield

$$3m^2 + 3mn - 2m - \tfrac{4}{3}n^2 = 0$$

$$3m^3 - 2m^2 + 2m^2n - \tfrac{7}{3}mn^2 - \tfrac{2}{3}n^2 + \tfrac{4}{3}n^3 = 0$$

which give

$$m = 0{\cdot}51, \qquad n = 1{\cdot}12. \tag{1.40}$$

The value of R_0 obtained is in accord with that given by equation (1.37). z_0 is a linear function with respect to time.

Figure 4 shows R_0 and z_0 given by equation (1.39) with $m = 0{\cdot}5$ and $n = 1{\cdot}0$ together with an experimental result of the nucleate boiling of saturated pure water on a heating surface of brass at atmospheric pressure measured photographically with a high-speed camera (6000 frames/s). Isshiki [5] reported a photographic study with the results similar to these. It should be noted from the figure that the relation of equation (1.39) with $m = 0{\cdot}5$ and $n = 1{\cdot}0$ could be valid not only for $t \approx t_d$ but for a more extended range of time.

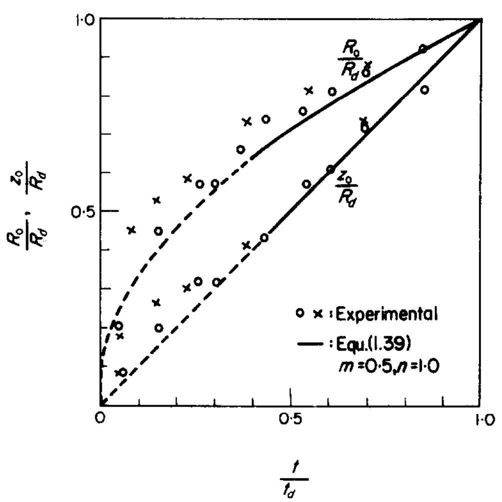

FIG. 4. Growth and departure curves.

GENERATION OF THE NEXT BUBBLE

When a bubble departs from the heating surface, the liquid surrounding the bubble flows into the space between the bubble and the heating surface. This motion of the liquid recomposes the temperature field in the vicinity of the heating surface, so that the next bubble appears on the heating surface when the field satisfies the condition responsible for the generation of an embryonic bubble. In this process of boiling, we make the following assumptions; (1) the acceleration of gravity is neglected, (2) the vapour inside and the liquid outside the bubble are of thermodynamic equilibrium so that the radius of the bubble remains constant at R_d after its departure from the heating surface, where R_d is the radius at the time of departure, (3) the temperature of the liquid immediately behind the bubble at the time of departure is T_s, (4) an embryonic bubble appears on the heating surface the instant that the amount of superheat of liquid, $T - T_s$, at the point of the spherical centre of a hypothetical embryonic bubble of radius R_e, $z_e = R_e \cos\varphi_e(\varphi_e$: contact angle), is enough to satisfy the Clausius–Clapeyron relation, (5) the temperature inside the embryonic bubble is equal to that of the liquid at the same position as the spherical centre of the bubble before its appearance, and (6) only the temperature field in the vicinity of the heating surface immediately behind the departed bubble ($r \approx 0$, $z \approx 0$) contributes to the generation of the bubble.

Let the distance between the spherical centre of bubble and the heating surface be z_0 and the velocity of the bubble be $\dot{z}_0$. If the effect of the viscosity of the liquid is neglected, the motion of the bubble has a velocity potential, ϕ, as

$$\phi = -\frac{R_d^3}{2}\left[1 + \frac{R_d^3}{(2z_0)^3}\right]\dot{z}_0\left\{\frac{z_0 + z}{[r^2 + (z_0 + z)^2]^{\frac{3}{2}}} + \right.$$
$$\left. + \frac{z_0 - z}{[r^2 + (z_0 - z)^2]^{\frac{3}{2}}}\right\}. \tag{2.1}$$

From equation (2.1), we obtain the components of the velocity of the bubble, u and w, in the r- and z-direction, respectively, in the vicinity of the heating surface immediately behind the bubble ($r \approx 0$, $z \ll z_0$), which are

$$u = -\beta r, \qquad w = 2\beta z \tag{2.2}$$

where

$$\beta = 3\left(\frac{R_d}{z_0}\right)^3 \left[1 + \tfrac{1}{8}\left(\frac{R_d}{z_0}\right)^3\right]\frac{\dot{z}_0}{z_0}. \tag{2.3}$$

It should be noted that the flow field described by equation (2.2) is similar to that in the vicinity of the stagnation point of a solid body immersed in an axisymmetrical inviscid flow. From the stagnation flow analogy, let

$$\begin{gathered} u = -rf'(z), \qquad w = 2f(z) \\ f(0) = f'(0) = 0, \qquad f'(\infty) = \beta \end{gathered} \tag{2.4}$$

satisfying the equation of continuity and the boundary conditions. Then, the equation of motion, introducing the viscous effect of the liquid, is reduced to

$$f'^2 - 2ff'' = \beta^2 - \nu f''' \tag{2.5}$$

where ν is the kinematic viscosity of the liquid. Solving equation (2.5) with the expansion procedure of $f(z)$ around $z = 0$ gives

$$w = 2\sqrt{(\beta\nu)}\left\{0{\cdot}656\left[\sqrt{\left(\frac{\beta}{\nu}\right)}z\right]^2 + 0{\cdot}1667\left[\sqrt{\left(\frac{\beta}{\nu}\right)}z\right]^3 + \ldots\right\}. \tag{2.6}$$

From equations (2.2) and (2.6), w can be expressed approximately for $z \ll z_0$ as

$$w = 2\beta z(1 - e^{-\delta z}), \qquad \delta = 0{\cdot}656\sqrt{(\beta/\nu)}. \tag{2.7}$$

The position of the spherical centre of the bubble, z_0, and its velocity, $\dot{z}_0$, can be obtained from the equation of motion with respect to the bubble, which is expressed by a procedure similar to that used in the derivation of equation (1.11) as

$$\tfrac{1}{2}\rho_0 V_0 \frac{d\dot{z}_0^2}{dt} = \frac{\partial}{\partial t}\int_{S_0} \frac{\rho}{2}\left(\phi\frac{\partial\phi}{\partial n}\right)_{S_0} dS_0 \tag{2.8}$$

where V_0 is the volume of bubble and the acceleration of gravity is not considered. Using equation (2.1) and the boundary condition at the interface of the bubble, $(\partial\phi/\partial n)_{S_0} = \dot{z}_0 \cos\varphi$ and since $\rho_0 \ll \rho$, we obtain from equation (2.8)

$$2\ddot{z}_0\left[1 + \tfrac{3}{8}\left(\frac{R_d}{z_0}\right)^3\right] - \tfrac{9}{8}\left(\frac{R_d}{z_0}\right)^3\frac{\dot{z}_0^2}{z_0} = 0\cdot \tag{2.9}$$

The solution of equation (2.9) in the expansion form with respect to time is

$$z_0 = R_d\left[1 + \frac{\dot{z}_d}{R_d}t + \tfrac{9}{44}\left(\frac{\dot{z}_d}{R_d}t\right)^2 - \tfrac{17}{34}\tfrac{9}{44}\left(\frac{\dot{z}_d}{R_d}t\right)^3 + \ldots\right. \tag{2.10}$$

where $\dot{z}_d$ is the velocity of the bubble at the time of departure, $t = 0$. Therefore, β becomes

$$\beta \approx 3\frac{\dot{z}_d}{R_d}\frac{1 + \frac{9}{22}\frac{\dot{z}_d}{R_d}t - \frac{9}{11}\frac{5}{44}\left(\frac{\dot{z}_d}{R_d}t\right)^2 + \ldots}{1 + 4\frac{\dot{z}_d}{R_d}t + \frac{229}{44}\left(\frac{\dot{z}_d}{R_d}t\right)^2 + \ldots} \tag{2.11}$$

which implies that the velocity component in the z-direction, w, decreases rapidly with increasing time.

From the assumption (6), which concerns the temperature field contributing to the generation of an embryonic bubble, we obtain the governing equation of the field as

$$\frac{\partial\theta}{\partial t} + w\frac{\partial\theta}{\partial z} = \alpha\frac{\partial^2\theta}{\partial z^2} \tag{2.12}$$

where T_w is the temperature of the heating surface, T_s that of the liquid at $z = \infty$, and

$$\theta = \frac{T - T_s}{T_w - T_s}.$$

The boundary conditions of equation (2.12) are provided by the assumptions (4) and (6) as

$$\theta(z,0) = 0, \qquad \theta(0,t) = 1. \tag{2.13}$$

Let us now turn our attention to the two extreme cases; the one in which w is so small that the conduction term (the right-hand side of equation (2.12)) is much greater than the convection term (the second term in the left-hand side) and the other in which w is so large that the conduction term can be neglected.

In the former case of small w, taking the solution of equation (2.12) with $w = 0$, $\theta^{(0)}(z,t)$,

$$\theta^{(0)}(z,t) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{z/[2\sqrt{(\alpha t)}]} \exp[-\xi^2]\,d\xi \quad (2.14)$$

as a basic solution, we can obtain the successive solutions, $\theta^{(1)}$, $\theta^{(2)}$, etc., from

$$\frac{\partial^2\theta^{(n)}}{\partial z^2} - \frac{1}{\alpha}\frac{\partial\theta^{(n)}}{\partial t} = w\frac{\partial\theta^{(n-1)}}{\partial z} \qquad n = 1, 2, 3, \ldots \quad (2.15)$$

Using the Green's function of equation (2.15), $G(z,t; z',t')$,

$$G(z,t; z',t') = 2(\sqrt{\pi})\,\alpha \exp\left[-\frac{1}{4\alpha}\frac{(z-z')^2}{(t-t')}\right]$$

to satisfy the boundary conditions (2.13), we obtain the solution of equation (2.15) as

$$\theta^{(1)}(z,t) = 4(\sqrt{\pi})\,\alpha \int_0^t dt' \left[\!\left[\int_{[-(1/2\sqrt{\alpha})][z/\sqrt{(t-t')}]}^{\infty} K\{z + 2\sqrt{[\alpha(t-t')]}\,z',t'\} \exp[-z'^2] \times dz' + \int_{(1/2\sqrt{\alpha})[z/\sqrt{(t-t')}]}^{\infty} K\{z - 2\sqrt{[\alpha(t-t')]}\,z', t'\} \exp[-z'^2]\,dz' \right]\!\right] + 1 - (2/\sqrt{\pi}) \int_0^{z/[2\sqrt{(\alpha t)}]} \exp[-\xi^2]\,d\xi \quad (2.16)$$

where

$$K(z,t) = \frac{1}{4\pi\sqrt{(\pi\alpha)}} w(z,t) \exp\left[-\frac{z^2}{4\alpha t}\right].$$

Denoting the first term in the right-hand side of equation (2.16) as $\Delta\theta^{(1)}(z,t)$ and substituting equation (2.7) into equation (2.16) yield

$$\Delta\theta^{(1)}(z,t) = \frac{1}{\pi\alpha}\int_0^t \beta(t')\frac{\sqrt{t'}}{t} z \exp\left[-\frac{1}{4\alpha}\frac{t'}{t^2}\cdot z^2\right] \times \{\sqrt{(4\pi\alpha)} + \delta \exp[\alpha\delta^2(t-t')]\,[4\alpha\sqrt{(t-t')} - 2\sqrt{(4\pi\alpha)}\,\alpha\delta(t-t')]\}\,dt'. \quad (2.17)$$

The second approximation, $\Delta\theta^{(2)}$, can be obtained similarly. β can be approximated from equation (2.11) as

$$\beta(t) = \beta_0\,e^{-\gamma t} \quad (2.18)$$

where $\beta_0 = 3\dot{z}_d/R_d$ and for small t

$$\gamma = 4\dot{z}_d/R_d. \quad (2.19)$$

From equations (2.17) and (2.18), we obtain

$$\Delta\theta^{(1)}(z,t) = \frac{2}{\sqrt{(\pi\alpha)}}\frac{z}{t}\beta_0 \int_0^t \sqrt{(t')} \exp\left[-\left(x + \frac{1}{4\alpha}\frac{z^2}{t^2}\right)t'\right] \left[\!\left[1 + \delta_0 \exp\left[-\frac{\gamma}{2}t' - \alpha\delta^2(t-t')\right]\left\{\sqrt{\frac{4\alpha}{\pi}}\sqrt{(t-t')} - 2\alpha\delta_0(t-t')\exp\left[-\frac{\gamma}{2}t'\right]\right\}\right]\!\right] dt' \quad (2.20)$$

where $\delta_0 = \delta_{t=0}$ [$= 0{\cdot}656\sqrt{(\beta_0/\nu)}$]. In the case of nucleate boiling of saturated pure water, the second term in the square brackets in the right-hand side of the above equation is negligibly small compared with the first term, so that the temperature field contributing to the generation of an embryonic bubble can be expressed approximately as

$$\theta(z,t) = 1 - \frac{1}{\sqrt{\pi}}\frac{z}{\sqrt{(\alpha t)}}\left[1 - (\sqrt{\pi})\,\beta_0\,\gamma^{-\frac{3}{2}}\frac{1}{\sqrt{t}}\right] + 0(z^2) \quad (2.21)$$

or, if we use equation (2.19),

$$\theta(z,t) = 1 - \frac{1}{\sqrt{\pi}}\frac{z}{\sqrt{(\alpha t)}}\left[1 - \frac{3\sqrt{\pi}}{8}\sqrt{\left(\frac{R_d}{\dot{z}_d}\right)}\frac{1}{\sqrt{t}}\right] + 0(z^2). \quad (2.21')$$

In the other case of large w where the convection term mainly governs the temperature field, using equation (2.2) for w and neglecting the viscous effect, we can solve equation (2.12) by the procedure of variable separation. Put

$$\theta(z,t) = 1 - (2/\sqrt{\pi}) \int_0^{z/\{2\sqrt{[\alpha g(t)]}\}} \exp[-\xi^2] \quad (2.22)$$

and $g(0) = 0$, so that θ satisfies the initial and boundary conditions (2.13). Then, equation (2.12) becomes

$$\frac{dg}{dt} - 4\beta g = 1$$

which yields

$$g(t) = \exp[\int_0^t 4\beta(t')\,dt'] \int_0^t \exp[-\int_0^{t'} 4\beta(t'') \times dt'']\,dt' \quad (2.23)$$

Using equation (2.18), equation (2.23) is reduced to

$$g(t) = \exp[-(4\beta_0/\gamma)\,e^{-\gamma t}] \int_0^t \exp[-(4\beta_0/\gamma) \times e^{-\gamma t'}]\,dt' = \exp[-(4\beta_0/\gamma)\,e^{-\gamma t}] \left\{t + \frac{1}{\gamma}\left[\frac{4\beta_0}{\gamma}(1 - e^{-\gamma t}) + \frac{(4\beta_0/\gamma)^2}{2 \cdot 2!} \times (1 - e^{-2\gamma t}) + \ldots\right]\right\} \quad (2.24)$$

or, with equation (2.19),

$$g(t) = \exp\left[-3\exp\left(-4\frac{\dot{z}_d}{R_d}t\right)\right]\int_0^t \times \exp\left[3\exp\left(-4\frac{\dot{z}_d}{R_d}t'\right)\right]dt'. \quad (2.24')$$

If the free energy of the system is kept constant before and after the appearance of an embryonic bubble, the Clausius–Clapeyron relation should hold between the pressure, p_e, and the temperature, T_e, within the bubble at its generation, as we assumed in the assumption (4);

$$\ln\frac{p_e}{p_\infty} = \frac{L}{R_g}\left(\frac{1}{T_s} - \frac{1}{T_e}\right).$$

Combining the above equation with equation (1.10) yields

$$\frac{2\sigma}{p_\infty R_e} = \frac{L}{R_g}\left(\frac{1}{T_s} - \frac{1}{T_e}\right). \quad (2.25)$$

With the help of the relation between R_e and z_e,

$$z_e = R_e \cos\varphi_e$$

equation (2.25) can be rewritten as

$$\theta_e = \frac{T_s}{T_w - T_s}\frac{z^* \cos\varphi_e}{z_e + z^* \cos\varphi_e} \quad (2.26)$$

where

$$z^* = \frac{2\sigma T_s R_g}{p_\infty L}.$$

From the assumption (4), an embryonic bubble cannot appear on the heating surface until equation (2.16) or (2.22) with $z = z_e$ satisfies equation (2.26). θ given by equation (2.16) or (2.22) and θ_e given by equation (2.26) decrease monotonically with increase of z_e, so that equations (2.26) and (2.16) or (2.22) with $z = z_e$ have more than one solution with respect to z_e (>0) for a given time t. Among these times t, for which there is at least one possible solution of $z_e(>0)$, the minimum is called the most favourable time, t_e, at which an embryonic bubble first appears. Substituting θ expressed by equation (2.16) or (2.22) to a first-order approximation with respect to z into equation (2.26) with $z = z_e$ yields

$$\left.\begin{aligned} & z_e^2 - \sqrt{(\pi\alpha t)}\left[\frac{1}{1-A} - \frac{z^*\cos\varphi_e}{\sqrt{(\pi\alpha t)}}\right]z_e \\ & \quad + \sqrt{(\pi\alpha t)}\frac{z^*\cos\varphi_e}{1-A}\frac{2T_s - T_w}{T_w - T_s} = 0 \\ & A \equiv (\sqrt{\pi})\,\beta_0\,\gamma^{-\frac{3}{2}}\,t^{-\frac{1}{2}} \\ & \text{or} \\ & z_e^2 - \sqrt{[\pi\alpha g(t)]}\left[1 - \frac{z^*\cos\varphi_e}{\sqrt{[\pi\alpha g(t)]}}\right]z_e \\ & \quad + \sqrt{[\pi\alpha g(t)]}\,z^*\cos\varphi_e\frac{2T_s - T_w}{T_w - T_s} = 0. \end{aligned}\right\} \quad (2.27)$$

Since t_e is the minimum value of t for which the above equation has real roots, z_e, we obtain

$$\left.\begin{aligned} &\sqrt{(\pi\alpha t_e)}\left[\frac{1}{1-A}-\frac{z^*\cos\varphi_e}{\sqrt{(\pi\alpha t_e)}}\right]^2 \\ &\quad=\frac{4z^*\cos\varphi_e}{1-A}\frac{2T_s-T_w}{T_w-T_s} \\ &z_e=\tfrac{1}{2}\sqrt{(\pi\alpha t_e)}\left[\frac{1}{1-A}-\frac{z^*\cos\varphi_e}{\sqrt{(\pi\alpha t_e)}}\right] \end{aligned}\right\} \quad (2.28)$$

or

$$\left.\begin{aligned} &\sqrt{[\pi\alpha g(t_e)]}\left[1-\frac{z^*\cos\varphi_e}{\sqrt{[\pi\alpha g(t_e)]}}\right]^2 \\ &\quad=4z^*\cos\varphi_e\frac{2T_s-T_w}{T_w-T_s} \\ &z_e=\tfrac{1}{2}\sqrt{[\pi\alpha g(t_e)]}\left[1-\frac{z^*\cos\varphi_e}{\sqrt{[\pi\alpha g(t_e)]}}\right]. \end{aligned}\right\} \quad (2.29)$$

For the nucleate boiling of saturated pure water, in which

$$z^*\cos\varphi_e \ll \sqrt{(\pi\alpha t_e)}, \qquad z^*\cos\varphi_e \ll \sqrt{[\pi\alpha g(t_e)]}$$

equations (2.28) and (2.29) become

$$\sqrt{(\pi\alpha t_e)}=4\left[1-(\sqrt{\pi})\frac{\beta_0}{\gamma}\frac{1}{\sqrt{(\gamma t_e)}}\right]z^* \times\cos\varphi_e\frac{2T_s-T_w}{T_w-T_s} \quad (2.30)$$

$$\sqrt{[\pi\alpha g(t_e)]}=4z^*\cos\varphi_e\frac{2T_s-T_w}{T_w-T_s}. \quad (2.31)$$

With the use of equation (2.19), equation (2.30) is furthermore reduced to

$$\sqrt{(\pi\alpha t_e)}=4\left(1-\frac{3\sqrt{\pi}}{8}\sqrt{\frac{R_d}{z_d t_e}}\right)z^* \times\cos\varphi_e\frac{2T_s-T_w}{T_w-T_s}. \quad (2.30')$$

From these equations, we obtain z_e, that is, $R_e(=z_e/\cos\varphi_e)$;

$$R_e=2z^*\frac{2T_s-T_w}{T_w-T_s} \quad (2.32)$$

which is illustrated in Fig. 5 for saturated pure water.

The growth rate of a bubble at the initial time, $(\dot{R}_0)_{t=0}=R_1$, can be obtained as follows. From equation (1.27),

$$q_e=\rho_e V_e c_p T_e c_1+L\rho_e V_e\left(\frac{V_1}{V_e}+Hc_1\right) -2\sigma\frac{V_e}{R_e}\left(\frac{V_1}{R_e}-\frac{R_1}{R_e}\right)+\sigma S_e\frac{S_1}{S_e} \quad (2.33)$$

where V_e and S_e are the volume and the area of the liquid–vapour interface of the embryonic

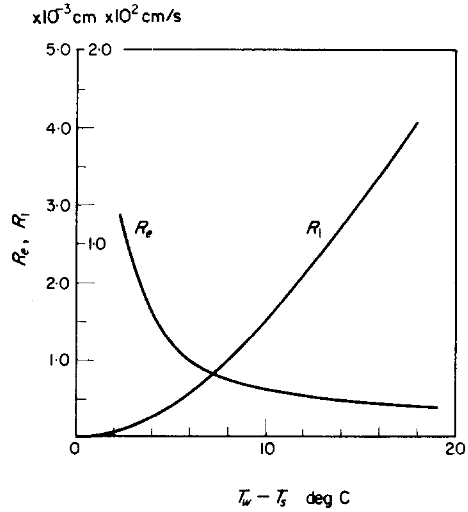

FIG. 5. Initial radius and growth rate vs. superheat.

bubble, respectively, and V_1 and S_1 are represented as

$$V_1=\frac{\pi}{3}(R_e-z_e)[2(2R_e+z_e)(R_1+z_1)+(R_e+z_e)(2R_1+z_1)]$$

$$S_1=2\pi[R_e(R_1+z_1)+R_1(R_e+z_e)].$$

The substitution of equations (1.26), (1.32) and (2.40′) into equation (2.33) yields

$$\frac{q_e}{\rho_e V_e c_p T_e}=\left[3\frac{L}{c_p T_e}+\frac{2\sigma}{R_e}\frac{1}{\rho_e c_p T_e} \times\frac{2-2\cos\varphi_e+\cos^2\varphi_e}{2+\cos\varphi_e-\cos^2\varphi_e} -\frac{2\sigma}{R_e}\frac{1}{p_e H}\left(1+\frac{L}{c_p T_e}H\right)\right]\frac{R_1}{R_e}. \quad (2.34)$$

If we assume that the temperature gradient at the liquid–vapour interface of a bubble is on an average equal to that at $z = z_e$, the amount of heat flow into the bubble through the interface at the embryonic instant can be written as

$$q_e = -\kappa\left(\frac{\partial T}{\partial z}\right)_{z_e,t_e} S_e \tag{2.35}$$

Obtaining $(\partial T/\partial z)_{z_e,t_e}$ from equation (2.16) or (2.22), we can get R_1 from equations (2.34) and (2.35) as

$$R_1 = \frac{\kappa}{L\rho_e}(T_w - T_s)\frac{1}{\sqrt{[\pi\alpha g(t_e)]}} \times \exp\left[-\frac{R_e^2\cos^2\varphi_e}{4\alpha g(t_e)}\right]\frac{2}{(1+\cos\varphi_e)(2-\cos\varphi_e)} \times\left\{1 + \tfrac{2}{3}\frac{\sigma}{\rho_e L R_e}\left[\frac{2 - 2\cos\varphi_e + \cos^2\varphi_e}{2 + \cos\varphi_e - \cos^2\varphi_e} - \left(H + \frac{c_p T_e}{L}\right)\right]\right\}. \tag{2.36}$$

For saturated pure water, the second term in the square brackets in the right-hand side of the above equation is negligibly small compared with unity. Then, R_1, can be written as

$$R_1 = \frac{2T_s\kappa}{L\rho_e}\frac{z^*}{R_e^2}\frac{1}{(1+\cos\varphi_e)(2-\cos\varphi_e)\cos\varphi_e}. \tag{2.36'}$$

Figure 5 shows R_1 for saturated pure water with $\cos\varphi_e = 0{\cdot}5$.

The initial velocity of a bubble, $(\dot{z}_0)_{t=0}$ $(= z_1)$, can be considered as follows. Suppose that the number of molecules in the liquid state is N at an instant and that at the next moment N_v molecules out of N turn into vapour so that N_l molecules remain in the liquid state. Let the free energy of a molecule in the liquid state be μ_l and that in the vapour state be μ_v. The change in the free energy of the system associated with evaporation, ΔG, is given by

$$\Delta G = N_v(\mu_v - \mu_l) + \sigma\, 2\pi R_0^2(1 + \cos\varphi_0) + (\sigma_{sv} - \sigma_{ls})\,\pi R_0^2\sin^2\varphi_0$$

where σ_{sv} and σ_{ls} are the surface tensions acting on the solid–vapour interface and the liquid–solid interface, respectively, and have the relation

$$\sigma_{sv} = \sigma_{ls} + \sigma\cos\varphi_0.$$

Using this relation and

$$N_v v_v = \frac{\pi}{3}R_0^3(1+\cos\varphi_0)^2(2-\cos\varphi_0) \qquad \left(v_v = \frac{V_0}{N_v}\right)$$

yields

$$\Delta G = \left[\frac{\pi}{3}(\mu_v - \mu_l)\frac{R_0^3}{v_v} + \pi R_0^2\sigma\right] \times (1+\cos\varphi_0)^2(2-\cos\varphi_0) \tag{2.37}$$

from which we obtain

$$\frac{d\Delta G}{dt} = \frac{dR_0}{dt}\left(\frac{\mu_v - \mu_l}{v_v}R_0^2 + 2R_0\sigma\right) \times \pi(1+\cos\varphi_0)^2(2-\cos\varphi_0) - \frac{d\varphi_0}{dt}\left(\frac{\mu_v-\mu_l}{v_v}R_0^3 + 3R_0^2\sigma\right)\pi\sin^3\varphi_0. \tag{2.38}$$

The assumption (4) concerning the condition of the generation of a bubble implies $(\partial\Delta G/\partial R_{t=0} = 0$ which gives

$$\frac{\mu_v - \mu_l}{v_v}R_e^2 + 2R_e\sigma = 0.$$

If we assume that, at the embryonic instant of a bubble, the process should develop so as to minimize the free energy of the system, we obtain

$$\left(\frac{d\Delta G}{dt}\right)_{t=0} = 0.$$

With the use of the above two relations, equation (2.38) becomes

$$\left(\frac{d\varphi}{dt}\right)_{t=0} = 0 \tag{2.39}$$

which means

$$(\dot{z}_0)_e = (\dot{R}_0)_e\cos\varphi_e \tag{2.40}$$

or

$$z_1 = R_1\cos\varphi_e. \tag{2.40'}$$

BUBBLE CYCLE

From equation (1.28) with $R_0 = z_0$, we can obtain the time interval from the appearance of an embryonic bubble to the departure of the bubble from the heating surface, t_d, and the radius of the bubble at the departure, R_d; namely, with the consideration of equation (1.33),

$$t_d = \tau_d \frac{R_e}{R_1}, \qquad R_d = \sigma_d R_e \tag{3.1}$$

where τ_d and σ_d are the functions of $\cos\varphi_e$. From equation (2.36), we obtain

$$t_d = \tau_d R_e \frac{L\rho_e}{2\kappa T_s z^*} f(\cos\varphi_e) \equiv \tau'_d R_e^3 \tag{3.2}$$

where $f(\cos\varphi_e)$ is a function of $\cos\varphi_e$. From equations (3.1) and (3.2), we obtain the relation between t_d and R_d as

$$\frac{R_d^3}{t_d} = \frac{\sigma_d^3}{\tau'_d}. \tag{3.3}$$

Since z_0 can be expressed from equation (1.40) as

$$z_0 = R_d \frac{t}{t_d}$$

$\dot{z}_d$ becomes from equation (3.3)

$$\dot{z}_d = \frac{\sigma_d^3}{\tau'_d}\frac{1}{R_d^2}. \tag{3.4}$$

This relation has been observed in the experimental study carried out by Isshiki [5] to hold approximately.

Concerning the radius of a bubble at departure, R_d, equation (1.38) gives

$$R_d = \frac{T_w - T_s}{\sqrt{(\pi\alpha)}}\frac{\kappa R_g T_s}{L\rho_\infty}\sqrt{t_d}.$$

If we put

$$t_d \propto R_e^n,$$

the above equation yields

$$R_d \propto R_e^{(n-2)/2}. \tag{3.5}$$

Since the usual results of experimental investigation show that the radius of a bubble at departure, R_d, increases as the temperature difference, $T_w - T_s$, decreases, that is, as its radius at the embryonic moment, R_e, increases, the value of n should be larger than 2. If we assume $R_d = \sigma_d R_e$ similar to equation (3.1), we obtain $n = 4$, that is, $R_d^4 \propto t_d$ [cf. equation (3.3)], which has a much greater deviation from the experimental results than equation (3.3) has, so that a more careful examination has to be directed equation (1.38).

The time interval from the departure of a bubble to the appearance of the next bubble, t_e, can be obtained from equation (2.28) or (2.29). When the temperature field is governed mainly by the conduction effect, we obtain from equation (2.28) or (2.30)

$$t_e = \frac{4}{\pi\alpha}\cos^{-2}\varphi_e \,.\, R_e^2 \equiv \tau'_e R_e^2. \tag{3.6}$$

On the other hand, when the field is dominated by the convection effect, from equation (2.24) with $\beta \approx \beta_{t=0}$, that is,

$$g(t) = \frac{1}{4\beta}(e^{4\beta t} - 1) \approx \frac{1}{4\beta}(4\beta t)^n$$

and $\beta \propto R_e^{-3}$, we obtain

$$t_e = \tau_e R_e^{3-(1/n)}$$

and for sufficiently large values of βt_e

$$t_e = \tau''_e R_e^3. \tag{3.7}$$

Finally, the complete cyclic period of the process of nucleation, growth, and departure can be found from equations (3.2) and (3.7) when the convection effect is associated with the motion of bubble as

$$\frac{R_d^3}{t_e + t_d} = \frac{\sigma_d^3}{\tau''_e + \tau'_d} \tag{3.8}$$

and from equations (3.2) and (3.6) when the conduction effect is predominant as

$$\frac{R_d^2}{t_e + t_d} = \frac{\sigma_d^2}{\tau'_e + R_e\tau'_d}. \tag{3.9}$$

Figure 6 shows experimental results of the relation between the radius of a bubble at departure and the cyclic period. It is shown from the figure that, as the radius of the bubble decreases, that is, at the higher superheat, the features of the experimental results approach

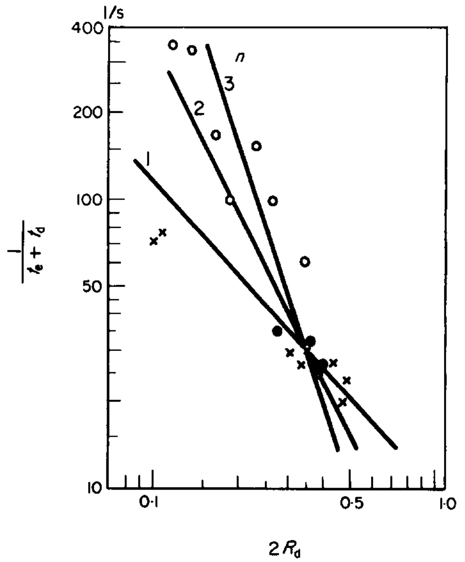

FIG. 6. Frequency of bubble cycle vs. departure radius.
○: reference [5]; ⊙: reference [8]; × reference [9]; $n = R_d^n/(t_e + t_d)$ = constant.

the relation expressed by equation (3.9) or (3.8), while Jakob's relation

$$\frac{R_d}{t_e + t_d} = \text{constant} \qquad (3.10)$$

holds satisfactorily for larger values of the radius, that is, at the lower superheat. For the larger values of the radius, the acceleration of gravity should be taken into consideration to account for the mechanism of boiling, as has been done by Han [10]. The mechanism of boiling considered from the fluid-dynamical point of view could be more predominant for the smaller values of the radius, i.e. at the higher superheat.

CONCLUSIONS

The process of nucleate boiling from a solid superheated surface, bubble generation, growth and departure, is studied fluid-dynamically and thermodynamically to account for its mechanism.

The asymmetry of the fluid-dynamical field associated with the existence of a solid heating surface causes a bubble to move away from the surface, with a nearly constant speed, while the surface area of the bubble increases linearly with respect to time. The time interval between the generation and departure of a bubble is proportional to the third power of the radius of the bubble at departure. Consideration of the velocity and temperature fields in the vicinity of the heating surface provides a relation between the period of the bubble cycle and the amount of superheat. The period is proportional to the third power of the radius of the bubble at departure when the fields interact strongly with each other, and to the second power when they do not. The radius of the bubble at departure is inversely proportional to the amount of superheat.

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Résumé—Le mécanisme de l'ébullition nucléée à partir d'une surface surchauffée est étudié du point de vue de la dynamique des fluides et de la thermodynamique.

La dissymétrie du champ dynamique associé à l'existence d'une surface chauffante provoque l'éloignement des bulles de la surface à une vitesse presque constante, tandis que la superficie des bulles croît linéairement en fonction du temps. L'intervalle de temps entre la formation d'une bulle et son détachement est proportionnel au cube de son rayon. Si l'on considère les champs de vitesse et de température au voisinage de la surface chauffante, on obtient une relation entre la période du cycle des bulles et la valeur de la surchauffe. La période est proportionnelle au cube du rayon de la bulle, lorsqu'il y aura une intéraction importante entre les champs, et au carré du rayon, lorsqu'il n'y en a pas. Le rayon d'une bulle est inversement proportionnel à la valeur de la surchauffe.

Zusammenfassung—Der Mechanismus des Blasensiedens an einer überhitzten Fläche wird flüssigkeitsdynamisch und thermodynamisch untersucht.

Die Asymmetrie des flüssigkeitsdynamischen Feldes zusammen mit der Heizfläche bewirken die Ablösung der Blasen von der Oberfläche mit nahezu konstanter Geschwindigkeit, während die Oberfläche der Blase hinsichtlich der Zeit linear zunimmt. Das Zeitintervall zwischen Blasenbildung und Abreissen ist proportional der dritten Potenz des Blasenradius. Die Betrachtung der Geschwindigkeits- und Temperaturfelder in der Umgebung der Heizfläche vermittelt eine Beziehung zwischen der Periode des Blasenwechsels und der Grösse der Überhitzung. Die Periode ist proportional der dritten Potenz des Blasenradius, wenn die Felder sich stark beeinflussen und proportional der zweiten Potenz, wenn sie es nicht tun. Der Blasenradius ist umgekehrt proportional der Grösse der Überhitzung.

Аннотация—Проведено термодинамическое и гидродинамическое исследование механизма пузырькового кипения на перегретой поверхности.

Наличие поверхности нагрева создает асимметрию гидродинамического поля, которая вызывает движение пузырьков от поверхности с почти постоянной скоростью, а площадь поверхности пузырков линейно возрастает со временем. Период времени между образованием пузырька и его отрывом пропорционален радиусу пузырька в третьей степени. Исследование полей скоростей и температур вблизи поверхности нагрева позволяет получить соотношение, связывающее время развития пузырька и величину перегрева. При сильном взаимодействии полей этот период пропорционален радиусу пузырька в третьей степени, и радиусу пузырька во второй степени при отсутствии взаимодействия полей. Радиус пузырька обратно пропорционален перегреву.